Freezing of spin dynamics and ω/T scaling in underdoped cuprates

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Abstract. The memory function approach to spin dynamics in doped antiferromagnetic insulator combined with the assumption of temperature independent static spin correlations and constant collective mode damping leads to ω/T scaling in a broad range. The theory involving a nonuniversal scaling parameter is used to analyze recent inelastic neutron scattering results for underdoped cuprates. Adopting modified damping function also the emerging central peak in low-doped cuprates at low temperatures can be explained within the same framework.

1. Introduction

It is by now experimentally well established that magnetic static and dynamical properties of high-T_c cuprates are quite anomalous. Early inelastic neutron scattering (INS) experiments on low-doped $La_{2-x}Sr_xCuO_4$ (LSCO) [1, 2] revealed that local, i.e., **q**-integrated dynamic spin response in the normal state (NS) exhibits anomalous ω/T scaling, not reflected in instantaneous spin-spin correlation length ξ_T which shows no significant T-dependence below room temperature. Subsequently similar behaviour has been found in a number of other compounds, i.e., in underdoped $YBaCu_3O_{6+x}$ (YBCO) and in Zn-doped YBCO [2]. More recent INS experiments on heavily underdoped (UD) cuprates, including Li-doped LSCO [3], YBCO [4, 5, 6, 7], and $Pr_{1-x}LaCe_xCuO_{4-\delta}$ (PLCCO) [8], confirm the universal features of anomalous NS spin dynamics so that ω/T scaling is found in a broad range both in **q**-integrated susceptibility $\chi''_L(\omega)$ [3, 5, 7] and in $\chi''_{\mathbf{q}}(\omega)$ at the commensurate AFM $\mathbf{q} = \mathbf{Q} = (\pi, \pi)$ [3, 5, 8].

Typically, $\chi''_{\mathbf{Q}}(\omega)$ is a Lorentzian with the characteristic relaxation rate scaling as $\Gamma = \alpha T$, but with a nonuniversal α [3, 5, 8]. Similarly, $\chi''_L(\omega, T) = \chi''_L(\omega, 0) f(\omega/T)$ has been used [2, 8, 6], with $f(x) = 2/\pi \arctan[A_1 x + A_2 x^3]$ and material dependent $A_{1,2}$. It has been also observed that at low $T < T_g$ some intensity is gradually transfered into a central peak (CP) [3, 4] whereas the inelastic response saturates. This *freezing* mechanism appears to be entirely dynamical in origin since ξ_T as well as the integrated intensity are unaffected by the crossover.

The present authors introduced a theory of spin dynamics in doped AFM [9] which describes the scaling behavior as a dynamical phenomenon based on two experimental observations: a) ξ_T is (almost) independent of T, and b) the system is metallic with finite spin collective-mode damping. Then the system close to AFM naturally exhibits ω/T scaling in a wide energy range, with saturation at low-enough T.

Our starting point in the analysis of recent INS experiments is the dynamical spin susceptibilty [9]

$$\chi_{\mathbf{q}}(\omega) = \frac{-\eta_{\mathbf{q}}}{\omega^2 + \omega M_{\mathbf{q}}(\omega) - \omega_{\mathbf{q}}^2}\,, \tag{1}$$

where the spin stiffness $\eta_{\mathbf{q}} \sim 2J\,(\sim 240$ meV) is only weakly $\mathbf{q}$-dependent (J is the exchange coupling), $\omega_{\mathbf{q}} = (\eta_{\mathbf{q}}/\chi^0_{\mathbf{q}})^{1/2}$ is an effective collective mode frequency, $\chi^0_{\mathbf{q}} = \chi_{\mathbf{q}}(\omega = 0)$ is the static susceptibility and $M_{\mathbf{q}}$ is (the complex) memory function containing information on collective mode damping $\gamma_{\mathbf{q}} = M''_{\mathbf{q}}(\omega)$. In the NS of cuprates low-frequency collective modes at $\mathbf{q} \sim \mathbf{Q}$ are generally overdamped so that $\gamma_{\mathbf{q}} > \omega_{\mathbf{q}}$. UD cuprates close to the AFM phase have low charge-carrier concentration but pronounced spin fluctuations whose dynamics is quite generally governed by the sum rule

$$\int_0^\infty \frac{d\omega}{\pi}\chi''_{\mathbf{q}}(\omega)\coth\frac{\omega}{2T} = C_{\mathbf{q}}\,, \tag{2}$$

where $C_{\mathbf{q}}$ is strongly peaked at $\mathbf{Q}$ with a characteristic width $\kappa_T = 1/\xi_T$. Moreover, the total sum rule is for a system with local magnetic moments (spin 1/2) given by $(1/N)\sum_{\mathbf{q}} C_{\mathbf{q}} = (1-c_h)/4$, where c_h is an effective (hole) doping.

While the formalism so far is very general, we now introduce approximations specific to UD cuprates [9]. INS experiments listed above indicate that within the NS the effective $\mathbf{q}$ width of $\chi''_{\mathbf{q}}(\omega)$, i.e., dynamical $\kappa(\omega)$, is only weakly T- and ω-dependent, even on entering the regime with the CP response [4]. Within further analysis we assume the commensurate AFM response at $\mathbf{Q}$ and the double-Lorentzian form $C_{\mathbf{q}} = C/[(\mathbf{q}-\mathbf{Q})^2 + \kappa_T^2]^2$ although qualitative results at low ω do not depend on a particular form of $C_{\mathbf{q}}$.

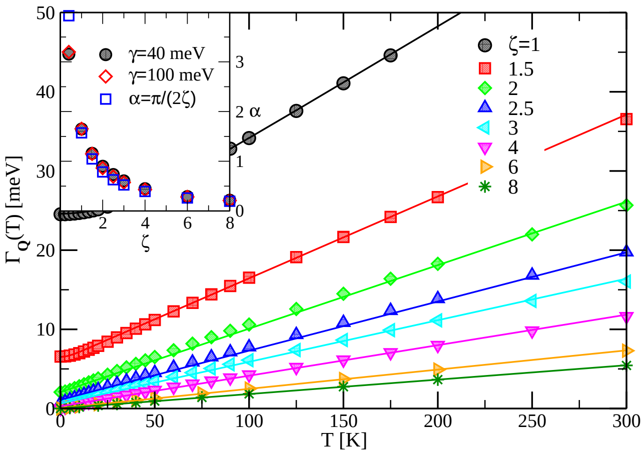


Figure 1. Relaxation rate $\Gamma_{\mathbf{Q}}$ vs. T for different parameters ζ and γ=100 meV. Inset: dependence of slope parameter α on ζ for two different γ. For comparison, an estimate for α is included.

2. Paramagnetic metal:

We also assume that the damping $\gamma_{\mathbf{q}}(\omega)$ is dominated by particle-hole excitations being only weakly $\mathbf{q}$ and ω dependent. Hence $\gamma_{\mathbf{q}}(\omega) \sim \gamma$, with γ a phenomenological parameter. The assumption of constant γ in Eq. (1) then leads to

$$\chi''_{\mathbf{q}}(\omega) \sim \chi^0_{\mathbf{q}}\frac{\omega\Gamma_{\mathbf{q}}}{\omega^2 + \Gamma^2_{\mathbf{q}}}, \quad \Gamma_{\mathbf{q}} = \frac{\eta}{\gamma\chi^0_{\mathbf{q}}}. \tag{3}$$

Note that recent INS data are fully consistent with this form which has been used to extract $\Gamma_{\mathbf{Q}}(T)$ [5].

We next exploit the sum rule, Eq.(2), to determine $\Gamma_{\mathbf{q}}$. As shown elsewhere [9, 10] $\Gamma_{\mathbf{Q}}$ is mainly determined by the parameter

$$\zeta = \pi\gamma C_{\mathbf{Q}}/(2\eta), \tag{4}$$

subject to $T \ll \gamma$ which is experimentally relevant. The results are presented in Fig. 1 for a range of $\zeta = 1-8$. While $\Gamma_{\mathbf{Q}}(0) = \Gamma^0_{\mathbf{Q}} \sim \gamma\exp(-2\zeta)$ [9, 10], for $T > \Gamma^0_{\mathbf{Q}}$ the variation is nearly linear

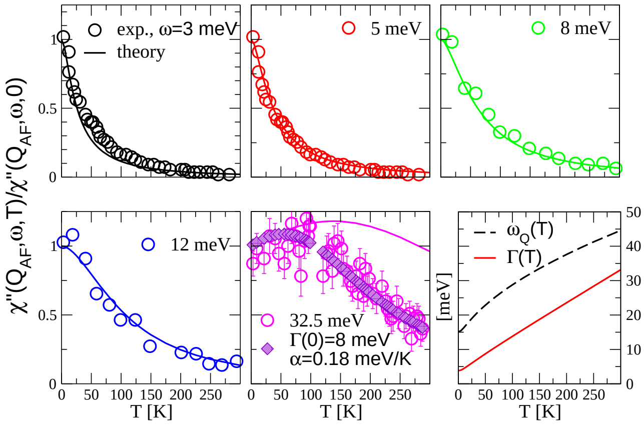
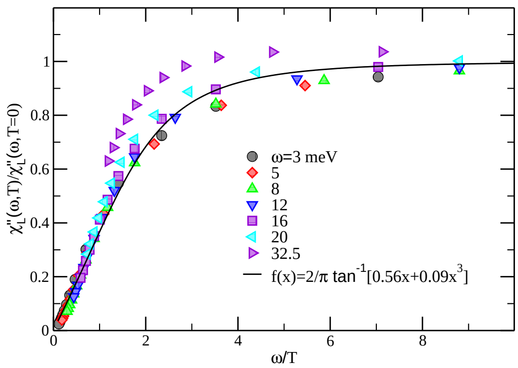


Figure 2. Temperature evolution of $\chi''_{\mathbf{Q}}(\omega, T)$ for UD YBCO with $x = 0.45$ [7] (symbols) compared with theoretical result, Eq. (3), with $\zeta = 1.8$ and γ=60 meV (lines).

Figure 3. Scaling of theoretical normalised $\chi''_L(\omega, T)$ based on ζ and γ as in Fig. (2). For comparison, the scaling function $f(x)$ as frequently used in fits to experiments is also plotted.



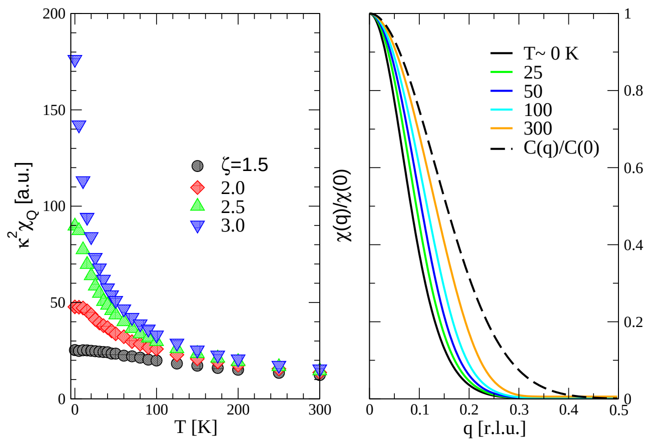


Figure 4. Left pannel: the temperature dependence of $\kappa^2\chi_{\mathbf{Q}}$ for several ζ as a function of T. Right pannel: T-dependence of $\chi_{\mathbf{q}}$ relative to a Lorentzian $C_{\mathbf{q}}$.

$\Gamma_{\mathbf{Q}} \sim \alpha T$ being a manifestation of the ω/T scaling. But α is not universal and depends on ζ (and weakly on γ). Then to leading order

$$\Gamma_{\mathbf{Q}}(T) \cong \max\{\frac{\pi T}{2\zeta}, \gamma \exp(-2\zeta)\}, \quad (5)$$

from which $\alpha = \pi/(2\zeta)$ is identified [10]. Note that the above $\Gamma_{\mathbf{Q}}$ leads to $\chi''_L(\omega)$ consistent with the "marginal Fermi liquid" model introduced by Varma et al. [11]. However, contrary to the usual assumption of proximity to a quantum critical point where $\xi_T \propto 1/T$, here $\xi_T \sim$ const.

Recently a number of INS experiments have been reported where $\chi''_{\mathbf{Q}}(\omega)$ can be well described by Eq.3, including linear-in-T behaviour of $\Gamma_{\mathbf{Q}}$ where, depending on material $\alpha \sim 0.18 - 0.75$ [3, 5, 8]. All these α require rather large ζ, which implies very low saturation $\Gamma^0_{\mathbf{Q}}$, whereas INS data for YBCO and LSCO indicate quite substantial $\Gamma^0_{\mathbf{Q}}$. However, saturation of $\Gamma_{\mathbf{Q}}$, setting in for $T < T_g$, is accompanied by simultaneous appearance of the CP which absorbs the 'missing' sum rule.

In Fig. 2 INS measurements by Hinkov et al. [6, 7] on UD YBCO with $x = 0.45$ [7] are presented together with theoretical curves were the only relevant parameter is $\zeta = 1.8$, while in Fig. 3 theoretical scaling function $\chi''_{\mathbf{Q}}(\omega, T)/\chi''_{\mathbf{Q}}(\omega, 0)$ for the same ζ (and γ) is plotted. The overall agreement between theory and experiment (Fig. 2) is quite satisfactory. The agreement is less satisfactory for $\omega = 32.5\,\mathrm{meV}$ and should be attributed to the breakdown of scaling since $\omega_{\mathbf{Q}} > \gamma/2$, as also evident in Fig. 3. Note that the *ad hoc* ansatz for $f(x)$ commonly used (with $A_2 = 0$) can be easily obtained assuming a Lorentzian dependence of $\chi^0_{\mathbf{q}}$ on $\mathbf{q}$. A simple

calculation yields $\arctan(A_1\omega/T)$ with $A_1 \sim 1/\alpha$, provided that $\kappa \ll 1$ but $\kappa^2\chi^0_{\mathbf{Q}} \sim$ const (see Fig. 4).

3. CP response:

The advantage of the memory-function formalism is that the emergence of the CP at $T < T_g$ in the spin response can be as well treated within the same framework. One has to assume that unlike in a paramagnet the mode damping $\gamma_{\mathbf{q}}$ is not constant but may acquire an additional low frequncy contribution. In particular we can take $\tilde{M}_{\mathbf{q}} \sim i\gamma - \delta^2/(\omega + i\lambda)$, with T-dependent δ and λ, which leads to $\chi''_{\mathbf{q}}(\omega)$ of the form used also to analyse experimental INS data for YBCO with $x = 0.35$ [4, 14].

For $\lambda \to 0$ but $\delta^2/\lambda \gg \gamma$ the modified $\tilde{M}_{\mathbf{q}}$ leads to two distinct energy scales and hence to two contributions to spin dynamics, i.e., the CP part $\chi^c_{\mathbf{q}}(\omega)$ and the regular contribution $\chi^r_{\mathbf{q}}(\omega)$,

$$\chi^c_{\mathbf{q}}(\omega) \sim \frac{\chi^0_{\mathbf{q}}\Gamma_c}{\Gamma_c - i\omega}, \ \chi^r_{\mathbf{q}}(\omega) \sim \frac{\chi^{r0}_{\mathbf{q}}\Gamma_r}{\Gamma_r - i\omega}, \ \mathbf{q} \sim \mathbf{Q}, \tag{6}$$

valid for $\omega < \lambda$ and $\lambda < \omega \ll \gamma$, respectively. Thus, below T_g new scales are set by $\Gamma_r = \Omega^2_{\mathbf{q}}/\gamma$ and $\Gamma_c = (\eta/\delta^2)\lambda/\chi^0_{\mathbf{q}}$, with $\chi^{r0}_{\mathbf{q}} = \eta/\Omega^2_{\mathbf{q}}$ and $\Omega^2_{\mathbf{q}} = \omega^2_{\mathbf{q}} + \delta^2$. If one assumes that δ saturates at low T, as is manifest by saturation of Γ_r [5], $\Gamma_c \propto \lambda/\chi^0_{\mathbf{q}}$ becomes the smallest energy scale, resulting in a quasielastic peak of width Γ_c. The saturation of Γ_r, although at present unclear physically, is responsible for the transfer of spectral weight, since $C^r_{\mathbf{q}} \sim C_{\mathbf{q}} - \pi T/(\gamma\Gamma_r)$ [10], which is again consistent with experiment on $x = 0.35$ YBCO [5].

4. Conclusions

The approach presented gives a consistent explanation of the ω/T scaling both in $\chi''_{\mathbf{q}}(\omega)$ as well as in $\chi''_L(\omega)$. It is based on two well established experimental facts: the overdamped nature of the response and the saturation of κ_T at low ω and T. The appearance of the CP for $T < T_g$ is easily incorporated into the formalism via the (almost) singular ω- and T-dependent damping $\tilde{M}_{\mathbf{q}}(\omega)$. However, the question as to the origin of CP remains to be settled.

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